

Tensor mass and particle number peak at the same location in the scalar-tensor gravity boson star models - an analytical proof

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Abstract

Recently in boson star models in framework of Brans-Dicke theory, three possible definitions of mass have been identified, all identical in general relativity, but different in scalar-tensor theories of gravity. It has been conjectured that it's the tensor mass which peaks, as a function of the central density, at the same location where the particle number takes its maximum. This is a very important property which is crucial for stability analysis via catastrophe theory. This conjecture has received some numerical support. Here we give an analytical proof of the conjecture in framework of the generalized scalar-tensor theory of gravity, confirming in this way the numerical calculations.

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Boson stars were first discussed by Kaup [1] and then by Ruffini and Bonazzola [2]. Boson stars in scalar-tensor theories of gravity have been investigated extensively by many researchers. The first model of a boson star in pure Brans-Dicke theory has been studied by Gunderson and Jensen [3]. Their work was generalized by Torres [4] who was studied boson stars in scalar-tensor theories with non-constant $\omega_{BD}(\Phi)$. More recently boson stars have been investigated in the papers by Torres et al. [5],[6] and in the paper by Comer and Shinkai [7]. In [5] boson

stars have been studied in connection with so-called gravitational memory [8], while their stability through cosmic history has been examined in [7] and [6]. Finally, the dynamical evolution of boson stars has been investigated in the paper by Balakrishna and Shinkai [9]. For more details we refer the reader to the most recent review on boson stars [10].

Here we consider complex scalar field boson stars in the most general scalar tensor theory of gravity with an action in Jordan frame

$$S = -\frac{1}{16\pi G_*} \int \sqrt{-\tilde{g}} \left(F(\Phi) \tilde{R} - H(\Phi) \tilde{g}^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \tilde{U}(\phi) \right) d^4x + \int \sqrt{-\tilde{g}} \left(\frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \Psi^+ \partial_\nu \Psi - W(\Psi^+ \Psi) \right) d^4x \quad (1)$$

where \tilde{R} is Ricci scalar curvature with respect to the space-time metric $\tilde{g}_{\mu\nu}$, G_* is the bare Newtonian constant and Φ is the gravitational Brans-Dicke scalar with potential term $\tilde{U}(\phi)$. Ψ is massive self-interacting complex scalar field with

$$W(\Psi^+ \Psi) = \frac{1}{2} m^2 \Psi^+ \Psi + \frac{1}{4} \lambda_* (\Psi^+ \Psi)^2.$$

Hereafter we will consider only static and spherically symmetric boson stars.

The explicit form of the action shows that we have $U(1)$ -invariant action under global gauge transformation $\Psi \rightarrow e^{ia} \Psi$, a being a constant. This global $U(1)$ -symmetry gives rise to the following conserved current

$$J^\mu = \frac{i}{2} \sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \left(\Psi \partial_\nu \Psi^+ - \Psi^+ \partial_\nu \Psi \right) \quad (2)$$

The conserved current leads to a conserved charge - total particle number making up the star

$$N = \int J^0 d^3x \quad (3)$$

Binding energy is then defined by

$$E_B = M_{Star} - mN \quad (4)$$

where m is the particle mass.

Now a problem arises: How to define the mass which appears in the expression for the binding energy?

In contrary to general relativity the definition of mass in scalar-tensor theories of gravity is quite subtle. This problem has been recently examined numerically in the work by Whinnett [11]. He has considered three possible mass definitions in Jordan frame, namely Schwarzschild mass M_S (i.e. ADM mass in Jordan frame), Keplerian mass M_{Kepler} and the tensor one M_T . As it has been shown numerically in [11] (see also [12]) these three masses differ significantly from each other

in the case $\omega_{BD} = -1$. Keplerian mass leads to positive binding energy which means that every boson star solution is in general potentially unstable. Contrary, Schwarzschild mass leads to negative binding energy suggesting that every solution is potentially stable even for large central densities $\rho = |\Psi(0)|^2$.

It should be noted that for large constant ω_{BD} (say $\omega_{BD} > 500$) the difference between the three masses is negligible. However, for arbitrary ¹ $\omega_{BD}(\phi)$ it's possible that the three masses may differ from each other significantly. This may occur in the early universe when the cosmological value Φ_∞ is sufficiently smaller than 1 [6]. Moreover, our numerical calculations show that for some physically relevant functions $\omega_{BD}(\phi)$ we may have $M_T - M_S \approx (0.15 - 0.20)M_T$. On the other hand the scalar tensor theories of gravity with $\omega_{BD} = -1$ better describe the early universe (see [13] and references therein). So, the case when ω_{BD} is not large, is also physically relevant. Therefore, when we study the boson stars in the early universe the mass choice is crucial.

It's the tensor mass which leads to physically acceptable picture. In [11] it's shown numerically that the tensor mass peaks at the same point as particle numbers - a very important property in general relativity [14]. This property is also crucial for the application of catastrophe theory to analyze the stability of the boson stars [15], [16].

In [11] and [6] the problem for analytical proof of the fact that it's the tensor mass which peaks at the same location as the particle number has been stated. The main purpose of this paper is to fill this gap.

In our opinion it's more convenient to work in Einstein frame given by

$$g_{\mu\nu} = F(\Phi)\tilde{g}_{\mu\nu} \quad (5)$$

In Einstein frame the action (2) takes the form

$$S = -\frac{1}{16\pi G_*} \int \sqrt{-g} (R - 2g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + U(\phi)) d^4x + \int \sqrt{-g} \left(\frac{1}{2} A^2(\phi) g^{\mu\nu} \partial_\mu \Psi^+ \partial_\nu \Psi - A^4(\phi) W(\Psi^+ \Psi) \right) d^4x \quad (6)$$

where R is the Ricci scalar curvature with respect to the metric $g_{\mu\nu}$, $U(\phi) = A^4(\phi)\tilde{U}(\Phi(\phi))$ and $A^2(\phi) = F^{-1}(\Phi(\phi))$ as ϕ is given by

$$\phi = \int d\xi \sqrt{\frac{3}{4} \left(\frac{d \ln(F(\xi))}{d\xi} \right)^2 + \frac{1}{2} \frac{H(\xi)}{F(\xi)}} \quad (7)$$

The action (6) leads to the following fields equations

$$G_\mu^\nu = \kappa_* T_\mu^\nu + 2\partial_\mu \phi \partial^\nu \phi - \partial^\sigma \phi \partial_\sigma \phi \delta_\mu^\nu + \frac{1}{2} U(\phi) \delta_\mu^\nu \quad (8)$$

¹We mean arbitrary $\omega_{BD}(\phi)$ for which the theory passes through all known gravitational experiments.

$$\begin{aligned}
\Box\phi + \frac{1}{4}U'(\phi) &= -\frac{\kappa_*}{2}\alpha(\phi)T \\
\Box\Psi + 2\alpha(\phi)\partial^\sigma\phi\partial_\sigma\Psi &= -2A^2(\phi)\frac{\partial W(\Psi^+\Psi)}{\partial\Psi^+} \\
\Box\Psi^+ + 2\alpha(\phi)\partial^\sigma\phi\partial_\sigma\Psi^+ &= -2A^2(\phi)\frac{\partial W(\Psi^+\Psi)}{\partial\Psi^+}
\end{aligned}$$

where \Box is d'Alembert operator in terms of the metric $g_{\mu\nu}$, $\kappa_* = 8\pi G_{*,\alpha}(\phi) = \frac{d}{d\phi}\ln(A(\phi))$ and T is the trace of the Einstein frame energy-momentum tensor of the complex scalar field given by

$$\begin{aligned}
T_\mu^\nu &= \frac{1}{2}A^2(\phi)\left(\partial_\mu\Psi^+\partial^\nu\Psi + \partial_\mu\Psi\partial^\nu\Psi^+\right) - \\
&\quad - \frac{1}{2}A^2(\phi)\left(\partial_\sigma\Psi^+\partial^\sigma\Psi - 2A^2(\phi)W(\Psi^+\Psi)\right)\delta_\mu^\nu
\end{aligned} \tag{9}$$

The conserved $U(1)$ -current in Einstein frame is

$$J^\mu = \frac{i}{2}A^2(\phi)\sqrt{-g}g^{\mu\nu}\left(\Psi\partial_\nu\Psi^+ - \Psi^+\partial_\nu\Psi\right) \tag{10}$$

As we have already mentioned we consider static and spherically symmetric boson stars i.e. space-time with a line element in Einstein frame

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2\left(d\theta^2 + \sin^2(\theta)d\varphi^2\right) \tag{11}$$

and complex scalar field in the form

$$\Psi = \sigma(r)e^{i\omega t} \tag{12}$$

where ω is real positive number and $\sigma(r)$ is a real function. It should be noted that the asymptotic behaviour of the field ϕ set the following tight constraint

$$0 < \omega < m \tag{13}$$

In this case the field equation system reduces to the following system of ordinary differential equations

$$\begin{aligned}
\lambda' &= \frac{1 - e^\lambda}{r} + \kappa_* e^\lambda r T_0^0 + r\phi'^2 + \frac{1}{2}U(\phi)e^\lambda r \\
\nu' &= \frac{e^\lambda - 1}{r} - \kappa_* e^\lambda r T_1^1 + r\phi'^2 - \frac{1}{2}U(\phi)e^\lambda r \\
\phi'' &= -\left(\frac{\nu' - \lambda'}{2} + \frac{2}{r}\right)\phi' + \frac{1}{4}U'(\phi)e^\lambda + \frac{\kappa_*}{2}\alpha(\phi)Te^\lambda \\
\sigma'' &= -\left(\frac{\nu' - \lambda'}{2} + \frac{2}{r}\right)\sigma' - \omega^2 e^{\nu-\lambda}\sigma - 2\alpha(\phi)\phi'\sigma' + 2A^2(\phi)e^\lambda W'(\sigma^2)\sigma
\end{aligned} \tag{14}$$

Here the components T_0^0 and T_1^1 are given correspondingly by

$$T_0^0 = \frac{1}{2}\omega^2 A^2(\phi)e^{-\nu}\sigma^2 + \frac{1}{2}A^2(\phi)e^{-\lambda}\sigma'^2 + A^4(\phi)W(\sigma^2) \quad (15)$$

$$T_1^1 = -\frac{1}{2}\omega^2 A^2(\phi)e^{-\nu}\sigma^2 - \frac{1}{2}A^2(\phi)e^{-\lambda}\sigma'^2 + A^4(\phi)W(\sigma^2) \quad (16)$$

The system (14) has to be solved at the following boundary conditions. We demand asymptotic flatness which means that $\nu(\infty) = 0$. On the other hand nonsingularity at the origin requires $\lambda(0) = 0$. Concerning ϕ , nonsingularity at the origin implies $\phi'(0) = 0$ while at infinity ϕ has to match the cosmological value $\phi(\infty) = \phi_\infty$. Nonsingularity of σ at the origin implies $\sigma'(0) = 0$. We require finite mass and therefore we put $\sigma(\infty) = 0$. In addition we have to give the central value $\sigma(0)$.

It's well known that the tensor mass is just the ADM mass in Einstein frame (we note that in Einstein frame all mass definitions coincide). Therefore we can write directly the explicit expression for the tensor mass using the first equation of (14), namely

$$M_T = \frac{1}{2G_*} \int_0^\infty dr r^2 \left(\kappa_* T_0^0 + e^{-\lambda} \phi'^2 + \frac{1}{2} U(\phi) \right) = \frac{1}{2G_*} \int_0^\infty dr r^2 \mathcal{D}(r) \quad (17)$$

as $\mathcal{D}(r)$ is defined by the expression itself. Respectively, the particle number is given by

$$N = 4\pi \int_0^\infty dr r^2 e^{\frac{\lambda}{2}} \left(\omega A^2(\phi) \sigma^2 e^{-\frac{\nu}{2}} \right) \quad (18)$$

When we are interested only in zeronodes solutions the boson star states are parameterized by the σ field central value $\sigma(0)$ or equivalently by $\rho = \sigma^2(0)$. Let's consider two infinitesimally nearby configurations parameterized by $\sigma(0)$ and $\sigma(0) + \delta\sigma(0)$. The corresponding variations of the tensor mass and particle number are

$$\delta M_T = \frac{1}{2G_*} \int_0^\infty dr r^2 \delta \mathcal{D} \quad (19)$$

$$\begin{aligned} \delta N &= 4\pi \int_0^\infty dr r^2 \delta \left(e^{\frac{\lambda}{2}} \omega A^2(\phi) \sigma^2 e^{-\frac{\nu}{2}} \right) = \\ &4\pi \int_0^\infty dr r^2 e^{\frac{\lambda}{2}} \delta \left(\omega A^2(\phi) \sigma^2 e^{-\frac{\nu}{2}} \right) + 4\pi \int_0^\infty dr r^2 \left(\omega A^2(\phi) \sigma^2 e^{-\frac{\nu}{2}} \right) \delta e^{\frac{\lambda}{2}} \end{aligned} \quad (20)$$

It's not difficult to obtain

$$\delta e^{\frac{\lambda}{2}} = \frac{1}{2r} e^{\frac{3}{2}\lambda} \int_0^r dr r^2 \delta \mathcal{D} \quad (21)$$

Now substituting (21) in (20) and after some algebra we have

$$\begin{aligned} \frac{\delta N}{4\pi} &= \int_0^\infty dr r^2 e^{\frac{\lambda}{2}} \delta \left(\omega A^2(\phi) \sigma^2 e^{-\frac{\nu}{2}} \right) + \\ &+ \frac{1}{2} \int_0^\infty dr r^2 \delta \mathcal{D} \int_r^\infty d\xi \xi \left(\omega A^2(\phi) \sigma^2 e^{-\frac{\nu}{2}} \right) e^{\frac{3}{2}\lambda} \end{aligned} \quad (22)$$

Taking into account the explicit form (17) of \mathcal{D} one obtains

$$\begin{aligned} \delta \left(\omega A^2(\phi) \sigma^2 e^{-\frac{\nu}{2}} \right) &= \frac{e^{\frac{\nu}{2}}}{\omega} \left(\delta \left(\frac{\mathcal{D}}{\kappa_*} \right) + \frac{1}{2} \omega^2 e^{-\nu} \delta \left(A^2(\phi) \sigma^2 \right) \right) \\ &- \frac{e^{\frac{\nu}{2}}}{\omega} \delta \left(\frac{1}{2} A^2(\phi) e^{-\lambda} \sigma'^2 + A^4(\phi) W(\sigma^2) + \frac{1}{\kappa_*} e^{-\lambda} \phi'^2 + \frac{1}{2\kappa_*} U(\phi) \right) \end{aligned} \quad (23)$$

In more detailed form the expression (23) is written as follows

$$\begin{aligned} \delta \left(\omega A^2(\phi) \sigma^2 e^{-\frac{\nu}{2}} \right) &= \frac{e^{\frac{\nu}{2}}}{\omega} \delta \left(\frac{\mathcal{D}}{\kappa_*} \right) - \frac{e^{\frac{\nu}{2}}}{\omega} \left(L_\phi \delta \phi + \frac{2}{\kappa_*} e^{-\lambda} \phi' \delta \phi' \right) \\ &- \frac{e^{\frac{\nu}{2}}}{\omega} \left(L_\sigma \delta \sigma + A^2(\phi) e^{-\lambda} \sigma' \delta \sigma' \right) - \frac{e^{\frac{\nu}{2}}}{\omega} \left(\frac{1}{2} A^2(\phi) \sigma'^2 + \frac{1}{\kappa_*} \phi'^2 \right) \delta e^{-\lambda} \end{aligned} \quad (24)$$

where L_ϕ and L_σ are given by

$$L_\phi = \alpha(\phi) \left(-\omega^2 e^{-\nu} A^2(\phi) \sigma^2 + A^2(\phi) e^{-\lambda} \sigma'^2 + 4A^4(\phi) W(\sigma^2) \right) + \frac{1}{2\kappa_*} U'(\phi) \quad (25)$$

$$L_\sigma = -\omega^2 e^{-\nu} A^2(\phi) \sigma + 2A^4(\phi) W'(\sigma^2) \sigma \quad (26)$$

Putting (24) in the first integral of (22), performing integration by parts and taking into account the third and the fourth equation of the system (14) one arrives at

$$\begin{aligned} \int_0^\infty dr r^2 e^{\frac{\lambda}{2}} \delta \left(\omega A^2(\phi) \sigma^2 e^{-\frac{\nu}{2}} \right) &= \int_0^\infty dr r^2 \frac{e^{\frac{1}{2}(\nu+\lambda)}}{\omega} \delta \left(\frac{\mathcal{D}}{\kappa_*} \right) - \\ &- \int_0^\infty dr r^2 \frac{e^{\frac{1}{2}(\nu+\lambda)}}{\omega} \left(\frac{1}{2} A^2(\phi) \sigma'^2 + \frac{1}{\kappa_*} \phi'^2 \right) \delta e^{-\lambda} \end{aligned} \quad (27)$$

Substituting now (27) in (22) as one uses that

$$\delta e^{-\lambda} = -\frac{1}{r} \int_0^r dr r^2 \delta \mathcal{D}$$

we have

$$\frac{\delta N}{4\pi} = \int_0^\infty dr r^2 \Lambda(r) \delta \left(\frac{\mathcal{D}}{\kappa_*} \right) \quad (28)$$

where $\Lambda(r)$ is the following expression

$$\Lambda(r) = \kappa_* \int_r^\infty d\xi \xi \left(\frac{1}{\omega} e^{\frac{1}{2}(\nu+\lambda)} \left(\frac{1}{2} A^2(\phi) \sigma'^2 + \frac{1}{\kappa_*} \phi'^2 \right) + \frac{1}{2} \omega A^2(\phi) \sigma^2 e^{-\frac{\nu}{2}} e^{\frac{3}{2}\lambda} \right) + \frac{1}{\omega} e^{\frac{1}{2}(\nu+\lambda)} \quad (29)$$

Using the first two equations of (14), it's not difficult one to show that Λ is actually a constant which turns out to be $\Lambda = \frac{1}{\omega}$. Therefore, we obtain finally

$$\frac{\delta N}{4\pi} = \frac{1}{\omega} \int_0^\infty dr r^2 \delta \frac{\mathcal{D}}{\kappa_*} \quad (30)$$

Comparing this expression with (19) we conclude that the following important relation holds

$$\delta M_T = \omega \delta N \quad (31)$$

The relation (31) may be rewritten in the form

$$\frac{\delta M_T}{\delta \rho} = \omega \frac{\delta N}{\delta \rho} \quad (32)$$

Taking into account that $0 < \omega < m$, it follows from (32) that if ρ_{crit} is a critical point for M_T (i.e. $\frac{\delta M_T}{\delta \rho} = 0$), then ρ_{crit} is also critical point for N .

Let M_T has a maximum at $\rho = \rho_{crit}$ ($\frac{\delta^2 M_T}{\delta \rho^2} < 0$). Then we obtain

$$\left(\frac{\delta^2 N}{\delta \rho^2} \right)_{crit} = \frac{1}{\omega_{crit}} \left(\frac{\delta^2 M_T}{\delta \rho^2} \right)_{crit} < 0 \quad (33)$$

which shows that N has a maximum at $\rho = \rho_{crit}$, too.

Therefore the tensor mass M_T and the particle number N peak, as functions of the central density, at the same location .

That the tensor mass and particle number peak at the same location results in a cusp in the bifurcation diagram M_T versus N . In infinitesimal small neighborhood of a cusp we may take expansions in powers of $(\rho - \rho_{crit})$

$$\begin{aligned} M_T &= M_{Tcrit} + \frac{1}{2} \left(\frac{\delta^2 M_T}{\delta \rho^2} \right)_{crit} (\rho - \rho_{crit})^2 + \frac{1}{6} \left(\frac{\delta^3 M_T}{\delta \rho^3} \right)_{crit} (\rho - \rho_{crit})^3 + O(4) \\ N &= N_{crit} + \frac{1}{2} \left(\frac{\delta^2 N}{\delta \rho^2} \right)_{crit} (\rho - \rho_{crit})^2 + \frac{1}{6} \left(\frac{\delta^3 N}{\delta \rho^3} \right)_{crit} (\rho - \rho_{crit})^3 + O(4) \end{aligned} \quad (34)$$

Now using (32), the coefficients $\left(\frac{\delta^2 M_T}{\delta \rho^2} \right)_{crit}$ and $\left(\frac{\delta^3 M_T}{\delta \rho^3} \right)_{crit}$ may be expressed by $N'' = \left(\frac{\delta^2 N}{\delta \rho^2} \right)_{crit}$ and $N''' = \left(\frac{\delta^3 N}{\delta \rho^3} \right)_{crit}$, and the result is

$$\left(\frac{\delta^2 M_T}{\delta \rho^2} \right)_{crit} = \omega_{crit} N''$$

$$\left(\frac{\delta^3 M_T}{\delta \rho^3}\right)_{crit} = \omega_{crit} N''' + 2\omega'_{crit} N''.$$

On the other hand, from the expansion for N we have

$$\rho - \rho_{crit} = \pm \left(\left(-\frac{2}{N''} \right) (N_{crit} - N) \right)^{\frac{1}{2}} + \left(\frac{N'''}{3N''^2} \right) (N_{crit} - N) + \dots$$

Substituting this expression in the expansion for M_T we obtain

$$M_T = M_{T_{crit}} + \omega_{crit}(N - N_{crit}) \mp \frac{1}{3}\omega'_{crit} \left(\frac{-2}{N''} \right)^{\frac{1}{2}} (N_{crit} - N)^{\frac{3}{2}} + O(2) \quad (35)$$

Let's denote by M_T^{up} and M_T^{low} correspondingly the tensor mass on the upper and lower branch of the curve $M_T(N)$. Then making use of (35) one obtains

$$M_T^{up} - M_T^{low} = -\frac{2}{3}\omega'_{crit} \left(\frac{-2}{N''} \right)^{\frac{1}{2}} (N_{crit} - N)^{\frac{3}{2}} \quad (36)$$

It's easy to see that for the binding energy we have the same relation

$$E_B^{up} - E_B^{low} = -\frac{2}{3}\omega'_{crit} \left(\frac{-2}{N''} \right)^{\frac{1}{2}} (N_{crit} - N)^{\frac{3}{2}} \quad (37)$$

These relations are scalar-tensor boson star versions of the similar relations in the fermion stars theory in pure general relativity [14]. It should be noted that such dependence $\sim (N_{crit} - N)^{\frac{3}{2}}$ is typical for catastrophe theory [17].

Conclusion

Scalar-tensor theories of gravity violate the strong equivalence principle. This results in the appearance of three different possible masses as a measure of the total energy of the boson star. The stability analysis of the boson stars requires that the mass and particle number peak, as functions of the central density, at the same location. In this article we have proved analytically that it's the tensor mass which possess the desirable property. Therefore, it's the tensor mass which should be taken as the physical mass, for example in the construction of the binding energy of the star. While the numerical calculations have been done for boson stars in pure Brans-Dicke theory of gravity, our proof holds for the most general scalar-tensor theory. Especially, our proof involves a potential term for the gravitational scalar Φ . This is important, because many scalar-tensor models of gravity involve such term. On the other hand the potential term may play significant role in the early universe and to influence the boson star formation and stability, although at present there aren't numerical investigations of boson stars in scalar-tensor theories with a potential term.

Finally, we believe that the result in this letter has a more general nature. It's the tensor mass which should be consider as the best candidate for the physical mass as a measure of the total energy of space-time in scalar-tensor theories of gravity.

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